Stat 112: Homework II

# 1 Linear Models

1. Six students were interviewed in 1957 in Moscow, U.S.S.R. about their statistics and English grades. The following information was obtained:

| | | | | | | |
|---|---|---|---|---|---|---|
| Statistics Grade | 70 | 92 | 80 | 74 | 65 | 83 |
| English Grade | 74 | 84 | 63 | 87 | 78 | 90 |

   (a) Plot the data. Let x be the statistics grade and y be the English grade. Assess the plot as to whether statistics grades are directly proportional to English grades.

   (b) Assert that a linear relation exits between the two grades. Write the linear model that describes your assertion. Be sure to state the probability distribution which you are assuming for $\epsilon$.

   (c) Compute the least squares estimates of $\beta_0$ and $\beta_1$.

   (d) Suppose everyone in STAT51 will get a statistics grade of 90. Estimate the sample average of the English grades that will be given at the end of the semester to the same class of STAT51 students.

   (e) Compute the 95% confidence interval of the estimate of that predicted sample average.

   (f) Forget about everyone else; predict your English grade based on the fitted least squares line, supposing that you will receive a statistics grade of 95.

   (g) Compute the 95% confidence interval of the estimate of your English grade.

   (h) Do you trust the model? Explain your answer.

2. You are planning to sell a used 2004 automobile and want to establish an asking price that is competitive. From a review of newspaper advertisements for used cars, you collect the following data where the asking price is in thousands of dollars and age is in years before the present:

$\sum_{i=1}^{12} x_i = 71 \quad \sum_{i=1}^{12} x_i^2 = 599 \quad \sum_{i=1}^{12} x_i y_i = 205.3 \quad \sum_{i=1}^{12} y_i = 60.5 \quad \sum_{i=1}^{12} y_i^2 = 441.87.$

   (a) Write the linear model:

   (b) $SS_{xx}$=

   (c) $SS_{xy}$=

   (d) Given that $\widehat{\beta}_0 = 10.089986$ and $\widehat{\beta}_1 = -0.853237$, enter the remaining values for $\widehat{E[y]}$ in the table.

   (e) Enter the remaining values for $\widehat{\epsilon}$ in the table.

| Age of car (x) | 1 | 1 | 2 | 3 | 3 | 4 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Asking Price (y) | 9.8 | 8.9 | 8.8 | 7.7 | 8.4 | 6.0 | 3.4 | 2.0 | 1.5 | 1.6 | 1.4 | 1.0 |
| $\widehat{E[y]}$ | 9.23 | | 8.38 | 7.53 | 7.53 | | 4.11 | | 2.41 | 1.56 | .70 | -0.14 |
| $\widehat{\epsilon}$ | 0.57 | -.33 | .42 | | | | | -1.26 | | | | 1.14 |

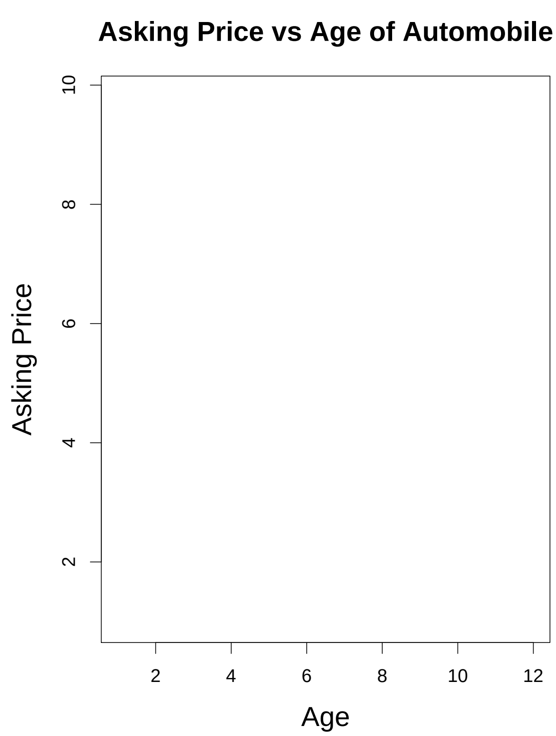
(f) Calculate $SSE$=

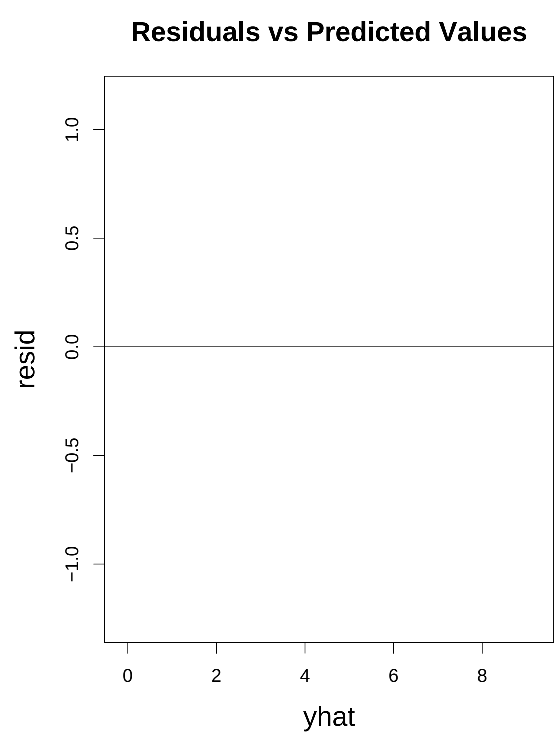
Based on your set of data and on the asserted linear model, what would be a good asking price, $y_p$, for your three year old car i.e.,

(g) Compute $\widehat{E[y_p]}$=

(h) Compute the 95% confidence interval of $E[\widehat{E[y_p]}]$.

(i) Plot the data.




(j) Plot the residuals versus predicted values.

(k) Do these two plots indicate that the model is a good model? Explain your answer.

(l) A common transformation which is used to rectify a bad plot of residuals versus predicted values is the square root transformation. The least squares fitted line for a square root transformation is: $\widehat{\sqrt{y_p}} = 3.3159605 - 0.2064453x_p$. Use this model to estimate the asking price, $y_p$, when the age of the car is $x_p = 3$.

# 2 Analysis of Variance

1. Have you ever wondered how tall Humpty Dumpty must have been before he fell from the wall? We have the tools to find his center of mass by constructing a confidence interval about $\mu$. Let us find his height. Fortunately, STAT51 students took pity on poor Humpty Dumpty by scraping-up what was left of him. The aggregated bits and pieces weighed 175 pounds (79275 grams). Of course, we know that Humpty Dumpty was not spherical but egg shaped. In order to correlate length (mm) with weight (grams) of an egg, the following set of data for 29 species of bird eggs ranging from the Hummingbird to the extinct Elephant Bird is given below:

| Bird | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Weight (grams) | 350 | .5 | 7.8 | 3.2 | 18.18 | 20.76 | 17.93 | 22.6 | 19.01 | 37.95 |
| log(weight), x, | 5.86 | -0.69 | 2.05 | 1.16 | 2.90 | 3.03 | 2.89 | 3.12 | 2.94 | 3.64 |
| Length (mm) | 113 | 13 | 28 | 21 | 30 | 32 | 31 | 35 | 32 | 41 |
| log(length), y | 4.73 | 2.56 | 3.33 | 3.04 | 3.40 | 3.47 | 3.43 | 3.56 | 3.47 | 3.71 |
| Bird | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| Weight (grams) | 7.55 | 4.47 | 3.78 | 5.02 | 3.09 | 3.01 | 5.38 | 7.1 | 4.54 | 4.41 |
| log(weight), x, | 2.02 | 1.50 | 1.33 | 1.61 | 1.13 | 1.10 | 1.68 | 1.96 | 1.51 | 1.48 |
| Length (mm) | 24 | 19 | 20 | 19 | 18 | 18 | 21 | 23 | 20 | 19 |
| log(length), y | 3.18 | 2.94 | 3.00 | 2.94 | 2.89 | 2.89 | 3.04 | 3.14 | 3.00 | 2.94 |
| Bird | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | |
| Weight (grams) | 11.95 | 7.34 | 8.62 | 4.19 | 2.72 | 4.43 | 6.30 | 1132.5 | 12231 | |
| log(weight), x, | 2.48 | 1.99 | 2.15 | 1.43 | 1.00 | 1.49 | 1.84 | 7.03 | 9.41 | |
| Length (mm) | 28 | 24 | 25 | 20 | 16 | 19 | 23 | 152.4 | 280 | |
| log(length), y | 3.33 | 3.18 | 3.22 | 3.00 | 2.77 | 2.94 | 3.14 | 5.03 | 5.63 | |

$\sum_{i=1}^{29} x_i = 71.0672$, $\sum_{i=1}^{29} x_i^2 = 283.7360$, $\sum_{i=1}^{29} x_i y_i = 274.7038$, $\sum_{i=1}^{29} y_i = 96.90692$, $\sum_{i=1}^{29} y_i^2 = 336.7251$, $SS_{xx} = 109.5792$, $SS_{xy} = 37.22434$, and $SSE = .2539898$.

Let w be the weight and l be the length of an egg. We know from our chemistry class that $l = \gamma w^{\frac{1}{3}}$ where $\gamma$ is related to density. Since Humpty Dumpty was not perfectly spherical, let us propose the following model: $l_i = \eta_0 w_i^{\eta_1}$ to relate his height and weight. What looks like a non-linear model can in fact be transformed into a linear model by taking logarithms, i.e. $log(l_i) = log(\eta_0) + \eta_1 log(w_i) + \epsilon_i$ where $log(x) = log_e(x) = ln(x)$. By doing so, an equivalent model for explaining the data can be written as a linear fixed effects model by replacing $log(l_i)$ with $y_i$, $log(\eta_0)$ with $\beta_0$, $\eta_1$ with $\beta_1$, and $log(w_i)$ with $x_i$ so that: $y_i = \beta_0 + \beta_1 x_i + \epsilon_i$ where $\epsilon_i \sim N(0, \sigma^2)$.

(a) Test the hypothesis that there is no inherent linear relation between the response variable and the explanatory variable at a level of significance, $\alpha = .05$.

(b) We are interested in finding Humpty Dumpty's height given that he weighed 79275 grams. Therefore, produce an estimate of $y_p$ using your fitted model.

(c) Of course, we want a confidence interval of our estimate. Compute the 95% CI for the particular value of $y_p$.

(d) Compute Humpty Dumpty's height, i.e. calculate $\widehat{height} = e^{\widehat{y}_p}$. Use the `exp` key on your calculator. The answer will be in millimeters. To convert to inches, divide your answer by 25.4.

(e) Make a plot of residuals versus predicted values.

(f) Do you approve of the model and trust the estimate? Explain.

2. Based on used car auctions, the final auction prices of 100 randomly selected three year Fleming XX500's and the respective odometer readings were recorded and plotted in a graph. Each of these cars was in good condition with the standard features like automatic transmission, radio, and air conditioning. The only essential difference about them was the odometer reading and final auction price. The following statistics were derived from the data in which y=price and x=odometer reading in thousands of miles: $\sum_{i=1}^{100} x_i = 3600.945$, $\sum_{i=1}^{100} x_i^2 = 133977.4$, $\sum_{i=1}^{100} x_i y_i = 19351920$, $\sum_{i=1}^{100} y_i = 541141$, $\sum y_i^2 = 2934770709$, $SS_{xx} = 4309.34$, and $SS_{xy} = -134269.3$, and SSE=2251362.

The plot of the data suggests the linear fixed effects model:$y_i = \beta_0 + \beta_i x_i + \epsilon_i$ where $\epsilon_i \sim N(0, \sigma^2)$

(a) Find $\widehat{\beta}_0 =$

(b) Find $\widehat{\beta}_1 =$

(c) Write the fitted equation for $\widehat{E[y_i]}$=

(d) Test the hypothesis, $H_0 : \beta_1 = 0$ vs $H_1 : \beta_1 \neq 0$ at a level of significance, $\alpha = .05$ $F_{1,98,.05}$=3.938111

| Source of Variation | df | Sum of Squares | Mean Sum of Squares | F statistic |
|---|---|---|---|---|
| Mean | | 2928335819 | | |
| Regression | | 4183528 | | |
| Residual Error | | | | |
| Total | | | | |

(e) What is $s^2 =$

(f) What is the estimated price on the average of a Fleming XX500 having 27000 miles on its odometer?

(g) The 95% confidence interval of the expected price of a Fleming XX500 which has 27000 miles on its odometer is (5387.035,5997.213). Would a trade-in value of \$5800 be a fair value to offer a customer wanting to buy a new car from you? Explain.

(h) Explain by referring to the plot of the data, the analysis of variance, and the residuals versus predicted value whether or not the model is good.