Homework 3

1. The random variable, X, has the following probability distribution (called also the probability mass function):

k	1	3	5	7	9
P(X=k)	.1	.2	.4	.2	.1

   Find:

   (a) P(X=5)=

   (b) P($X \leq 3$)=

   (c) P($X \geq 3$)=

   (d) P($3 \leq X \leq 7$)=

   (e) P($X \leq 9$)=

2. A deck of four cards are numbered 1, 2, 3, and 4, and they are placed face down on a table. Two cards are flipped over to reveal their numbers. The order of the cards is irrelevant. Assume that the outcomes are equally likely. Let X be the random variable such that X is the sum of the numbers which are visible.

   (a) List all possible outcomes. There are $\binom{4}{2} = 6$ of them.

   $$\Omega = \left\{ \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \right\}$$

   (b) List the five possible values of X.

   (c) List the elements comprising the event $A = \{\omega \in \Omega | X = 4\}$.

   (d) List the elements comprising the event $B = \{\omega \in \Omega | X = 6\}$.

   (e) Compute:

      i. P(A)=

      ii. P(B)=

      iii. P($X > 4$)=

      iv. P($3 \leq X < 6$)=

      v. P(5X+3=18)=

      vi. P(X is even)=

3. The probabilities of each outcome is listed in the following table.

Warehouse	Shaft		
	1	2	3
1	.41	.06	0
2	.10	.15	.04
3	.11	.07	.06

There are three warehouses from which a shaft is drawn. Let X be a random variable that designates a warehouse, that is, X=1, 2, or 3. Let Y be a random variable that describes the stiffness of a shaft. Let Y=1 represent a regular shaft; Y=2 represent a stiff shaft; Y=3 represent an extra stiff shaft.

(a) List all possible outcomes that constitute the sample space. Remember to use set notation.

(b) Let the event $C = \{\omega \in \Omega | X(\omega) = 3\}$. List the elements of C.

(c) List the elements in $D = \{\omega \in \Omega | Y(\omega) = 1\}$.

(d) List the elements in $G = \{\omega \in \Omega | X(\omega) = 3 \quad and \quad Y(\omega) = 2\}$.

(e) List the elements in $H = \{\omega \in \Omega | 1 \leq X(\omega) \leq 3 \quad and \quad Y(\omega) > 2\}$.

(f) Find:

i. P(D)=

ii. P(G)=

iii. P(H)=

iv. P(3X-2=4)=

v. P(X+Y=3)=

vi. P($4 \leq XY \leq 6$)=

4. Four fair coins are tossed simultaneously. Let T represent the outcome of getting a tail, and l et H represent the outcome of getting a head. Let A be the event that three and only three heads appear, and let B be the event that at least one tail appears.

(a) Write all the elements of $A \cap B^c$ using correct notation.

(b) What is P(B)?

5. Two fair dice are tossed. Find the probability that a three appears on either face.

6. From an urn which contains 5 red balls and 3 white balls, one ball is drawn at random. What is the probability that the selected ball is white?

7. The outcomes of two variables are (Low, Medium, High) and (On, Off), respectively. An experiment is conducted in which the outcomes of each of the two variables are observed. The probabilities associated with each of the six possible outcomes pairs are given in the accompanying table.

	Low	Medium	High
On	.50	.10	.05
Off	.25	.07	.03

Given that $A : \{On\}$, $B : \{Medium\ or\ On\}$, $C : \{Off\ and\ Low\}$, $D : \{High\}$Find:

(a) P(A)=

(b) $P(A \cup B)$=

(c) $P(C \cup D) =$

8. For two events, A and B, P(A)=.5 and P(B)=.2, if A and B are disjoint, then find:

   (a) $P(A \cap B)$=

   (b) $P(A|B)$=

   (c) $P(A \cup B)$=

9. A manufactured product especially an expensive one which is sold under warranty is generally more desirable than a competitor's product which does not carry a warranty. A warranty is not free, however, as the cost of labor, of replacement, and of customer service for maintaining effective marketing increase the company's overhead. Denote D to be a defective product which has become a part of the inventory of a wholesaler so that the inventory, $I = \{t,\ u,\ v,\ D,\ w\}$ where t, u, v, and w are good products.

   (a) A retailer orders two products from the wholesaler. List all possible shipments of two items: $\Omega = \Big\{ \qquad\qquad\qquad\qquad\qquad\qquad \Big\}$

   (b) Let E be the event that a retailer receives a defective product. List the elements of $E = \Big\{ \qquad\qquad\qquad\qquad\qquad\qquad \Big\}$

   (c) Given that the outcomes are equally likely, find P(E)=

   (d) Let F be the event of receiving a defective product when a shipment contains four items. Given that the outcomes are equally likely, find P(F)=

10. Three fair coins are tossed at a time so that the sample space consists of eight elements:

    $\Omega = \Big\{TTT\ \ TTH\ \ THT\ \ HTT\ \ THH\ \ HTH\ \ HHT\ \ HHH\Big\}$

    The events M and N are defined as follows:

    $M = \{\omega \in \Omega\ such\ that\ at\ most\ one\ head\ is\ observed\}$

    $N = \{\omega \in \Omega\ such\ that\ the\ number\ of\ heads\ observed\ is\ odd\}$

    (a) List (use correct notation)the elements of M=

(b) List the elements of N=

(c) List the elements of $M \cap N$=

(d) Find $P(M \cap N) =$

(e) Find $P(M) =$

(f) Are the events M and N mutually exclusive? Why?