Homework 4

1. The probability that Gloria will answer her telephone is $\frac{3}{5}$ and will not answer her telephone is $\frac{2}{5}$. Let X be a random variable such that X=1 if the telephone is answered and X=0 if it is not answered.

   (a) List the elements of the sample space, Ω.

   (b) List the elements that comprise the event, $\{\omega \in \Omega | X = 1\}$.

   (c) What is P(X=1)?

2. Suppose Gloria, Ann, Sue, and Kathy will each independently answer their telephones with probability $\frac{3}{5}$ and will not answer their telephones with probability $\frac{2}{5}$. Let X be a random variable such that X=the number of telephones that are answered. The possible values of X are 0, 1, 2, 3, and 4. Find P(X=0), P(X=1), P(X=2), P(X=3), and P(X=4).

   (a) Method I: Enumerate the possible elements in each event. Compute the probability of each outcome in an event and use the theorem that the probability of an event is the sum of the probabilities of each outcome because the outcomes are disjoint. (Hint: Let A=answer, N=no answer. One possible outcome is AANN corresponding to Gloria and Ann answering their telephones and Sue and Kathy not answering their telephones. By independence the probability of the outcome AANN is $\frac{3}{5}\frac{3}{5}\frac{2}{5}\frac{2}{5} = \frac{36}{625}$. There are 16 elements in the sample space).

   (b) Method II: Let X follow a binomial distribution. Use the formula for the binomial distribution with n=4 and p=$\frac{3}{5}$. That is,
   $P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$.

      i. Find $P(X \leq 2)$.
      ii. Find $P(X > 3)$.
      iii. Find E[X].
      iv. Find var(X).

3. Let X be a Bernoulli random variable such that P(X=1)=.7 and P(X=0)=.3. This random variable might be associated with the events of getting rain or sunshine tomorrow.

   (a) Find E[X].

   (b) Find var(X).

4. Let Y be a uniform random variable such that Y equals the number on the face of a fair die which is rolled: Y=1, 2, 3, 4, 5, and 6.

   (a) Find P(Y=4).

   (b) Find E[Y].

(c) Find var(Y).

5. Let U be a binomial, b(8, .4), random variable such that U is the number that appears on a simple slot machine.

   (a) Find E[U].
   (b) Find var(U).
   (c) Find P($U \leq 6$), P($U \leq 4$), P(U=5), and P($U \geq 7$).

6. Rely on problems 3, 4 , and 5. Consider this strategy: If it is sunny tomorrow, then roll a die. If it is rainy, then play the slot machine. In terms of our random variables:

   If X=1, then roll die.

   If X=0, then play slot machine.

   Under this strategy:

   (a) Find P($Y = 1|X = 1$), P($Y = 2|X = 0$), P($U = 5|X = 0$), and P($U \leq 4|X = 1$).
   (b) Find P(Y=1). (Hint: Use the theorem of decomposing an event. Recall that for events A and B, $P(A) = P(A|B)P(B) + P(A|B^c)P(B^c)$; for our problem, let A=$\{\omega \in \Omega|Y = 1\}$ and $B = \{\omega \in \Omega|X = 1\}$; therefore,
   P(Y=1)=$P(Y = 1|X = 1)P(X = 1) + P(Y = 1|X = 0)P(X = 0)$.)
   (c) Find P(U=1).

7. Let W, the amount of someone's lottery prize, be a random variable such that:

   W=$1000 with probability .1

   W=$50 with probability .3

   W=$0 with probability .6

   (a) Find E[W].
   (b) Find var(W).

8. Taxes, denoted by T, must be paid on lottery winnings according to the following schedule.

   If W=$1000, then T=$200.

   If W=$50, then T=$5.

   If W=$0, then T=$0.

   Under these conditions:

   (a) Find P($T = 200|W = 1000$), P($T = 200|W = 50$), P($T = 200|W = 0$)

(b) Find P(T=200). (Hint: Use a more general form of decomposing an event. That is, let A=$\{\omega \in \Omega | T = 200\}$, $B_1 = \{\omega \in \Omega | W = 1000\}$, $B_2 = \{\omega \in \Omega | W = 50\}$, and $B_3 = \{\omega \in \Omega | W = 0\}$, then
P(T=200)=P(A)=P(A$|B_1$)P(B_1)+P(A$|B_2$)P(B_2)+P($A|B_3$)P(B_3).
(Notice that the events B_1, B_2, and B_3 are disjoint.)

(c) Use this method to find P(T=5) and P(T=0).
(Answers: P(T=200)=.1, P(T=5)=.3, and P(T=0)=.6).

(d) Find E[T] and var(T).

9. A particular automatic sprinkler system for high-rise apartment buildings has two activation devices for each sprinkler head. One type has a reliability of .91 and the other which operates independently of the first has a reliability of .87. What is the probability that the sprinkler head will be activated?

10. The age distribution for employees of a highly successful company is shown below. An employee is selected at random from this population. Define the random variable, X, to be the age of an employee. Assume that the age distribution shown below is the same as the probability distribution of X.

Age	20	21	22	23	24	25	26	27	28	29	30	31	32	33
Proportion	.02	.04	.05	.07	.04	.02	.07	.02	.11	.07	.09	.13	.15	.12

(a) Find $P(X \geq 30) =$

(b) Find $P(26 < X \leq 30) =$

(c) Find the age, c, such that $P(X \leq c) = .22$

11. Let Y be a discrete uniformly distributed random variable such that the possible values of Y are: -2, -1, 0, 1, 2. Find:

(a) P(Y=-2)=

(b) E[Y]=

(c) var(Y)=

12. A survey of one thousand randomly selected people who shopped at a shopping mall were asked about the items which they purchased. The results of the interviews follow.

Sex	Items Purchased			
	Clothing	Shoes	Other	Total
Male	75	25	150	
Female	350	230	170	
Total				

(a) Find the probability that a shopper is a woman and buys shoes.

(b) Let X be a random variable such that $X(\omega) = 1$ if ω is male and $X(\omega) = 0$ if ω is female. Find E[X]=

(c) Important information was not given like the precise definition of the population. Suppose that the survey was conducted at the Georgetown Mall on the morning of 24 December 1959. Is your answer for the probability that a woman will purchase shoes valid for shopping behavior at the Georgetown Mall on 1 October 2007? Explain.

13. A fair eight sided (octahedral) die is tossed with a fair coin with a number 1 inscribed on one side of the coin and the number 0 inscribed on the other side. Let X be the random variable such that X is the sum of the numbers which appear on top after the die and coin are tossed.

    (a) List all possible outcomes.
    $\Omega = \Big\{ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \Big\}$

    (b) List the elements comprising the event $A = \{\omega \in \Omega | X(\omega) \leq 4\}$.
    $A = \Big\{ \qquad\qquad\qquad\qquad \Big\}$

    (c) Find: $P(A) =$

    (d) Let W equal the number on top of the die and let Z equal the number on the coin. Find $P(W = 5 | Z = 1) =$

    (e) Are the events, $\{\omega \in \Omega | W(\omega) = 5\}$ and $\{\omega \in \Omega | Z(\omega) = 1\}$ independent? Explain.

14. Suppose a poll of 20 employees is taken in a large company . The purpose is to determine, x, the number who favor unionization. Suppose that 60% of all the company's employees favor unionization.

    (a) What distribution does x follow? (Be specific.)

    (b) What is the expected number of employees who will support the creation of a union?