Homework 5

1. Each of ten multiple choice questions on a quiz has five possible answers of which only one is correct. Five or more questions must be answered correctly in order to pass the quiz.

   (a) What is the probability that someone can pass the quiz without reading a question? (Ans.: 0.0327935)

   (b) Suppose our sly student received a tip from somebody who knew for a fact that the instructor purposely never chose the third answer to be correct. Now, what is the probability that he can pass the quiz without reading a question? (Ans.: 0.0781269)

   (c) Suppose the quiz consisted of ten true-false questions. What is the probability that some one can pass the quiz without reading a question? (Ans.: 0.6230469)

   (d) What is the expected number of correct answers for each of the above situations?

2. Six apples were picked from an apple tree and were weighed. Their weights in grams are: 200, 250, 220, 230, 225, 210.

   (a) Find $\bar{x}$ and s^2.

   (b) Let W be the weight of an apple. Is W a discrete or continuous random variable? Why?

   (c) Let us assert that $W \sim N(\bar{x}, s^2)$. Find $P(W \leq 250)$.

   (d) What are E[W] and var(W)?

   (e) Someone picked another apple from the same tree and ate it. Later he bragged to his buddies that it was the biggest apple on the apple tree saying that it must have weighed 275 grams. What is the probability that an apple of that size or larger could have come from that tree? (Ans.: 0.00116806)

3. The average grade for a BIG statistics examination is 82 with a standard deviation of 13. Suppose that the instructor wants, in the name of competition, to grade the exam on a curve so that 5% of the class will flunk. After all, only the best survive, you know; too bad for those who diligently do their homework but have bright classmates. Be that as it may, what is the passing grade? (Ans.: 60.6169)

4. Let $T \sim U(0, 10)$. Find $P(T \leq 1)$, $P(T \geq 7)$, find c such that $P(T \leq c) = .6$, $P(-2 \leq T \leq 2)$, $P(T \leq 3 \; or \; T > 8)$, E[T], and var(T).

5. A researcher must pick an apple from either of two apple trees which are named 1 and 2. The weights of the apples in grams from each tree are distributed as follows: $W_1 \sim N(220, 100)$, and $W_2 \sim N(275, 225)$, respectively. He selects a tree at random by flipping a loaded coin on which a head appears according to a Bernoulli random variable, $U \sim b(1, .3)$. Before entering the orchard, it was decided by him to select tree 1 if a head appears. What is the probability that an apple weighing less than 230 grams will be picked? (Hint: Decompose the event into smaller pieces. Ans.: 0.2533)

6. Let Y_1, Y_2, Y_3, and Y_4 be independent random variables each of which follows the binomial distribution, b(4,$\frac{7}{12}$) and denote, $\bar{Y} = \frac{Y_1+Y_2+Y_3+Y_4}{4}$. Find: E[$Y_1$], var($Y_3$), E[$\bar{Y}$], and var($\bar{Y}$).

7. A random sample consisting of 5 elements is drawn from a population which the binomial distribution, b(7,.5), adequately describes. Find:

   (a) the population mean, μ=

   (b) the population variance, σ^2=

   (c) the expected value of the sample mean, $E[\bar{x}]$=

   (d) the variance of the sample mean, $var(\bar{x})$=

8. A random sample consisting of 9 elements is drawn from a population which the normal distribution, N(7,16), adequately describes. Find:

   (a) the population mean, μ=

   (b) the population variance, σ^2=

   (c) E[$\bar{x}$]=

   (d) var($\bar{x}$)=

9. Still using the previous distribution, let $\bar{x} \sim N(\xi, \varsigma^2)$, what is the value for:

   (a) ξ=

   (b) ς^2=

   (c) If $\bar{x} = 7.4$, find the z-score of $\bar{x}$. z=

   (d) Find P($\bar{x} \leq 7.4$)=

   (e) Find P($6.6 \leq \bar{x} \leq 8.6$)=

   (f) Find P$\left(\bar{x} \leq E[\bar{x}] + 3\sqrt{var(\bar{x})}\right)$=

10. Parakeets (*Melopsittacus undulatus* also called budgerigars) are used in the Laboratory of Comparative Psychoacoustics at the University Maryland - College Park. The greatest auditory sensitivity of the parakeet corresponds closely to the frequencies found in human speech. After parakeets are trained, they are used in experiments instead of people for the purpose of studying the effects of noise on hearing and speaking. Suppose at a background noise level of 70 decibels (e.g. *fortissimo* on a trombone) the measurements of the maximum distance in meters of song detection from 10 parakeets are $\{18,\ 20,\ 16,\ 18,\ 21,\ 18,\ 20,\ 19,\ 20,\ 17\}$. Denote D to be the distance and assume that it is normally distributed as $N(\bar{x}, s^2)$. Find d_{70} which represents the maximum threshold distance at 70 decibels such that $P(D \leq d_{70})$=.95. (Ans.: $d_{70} = 21.27752$ meters. Never mind the excessive number of significant digits used in the answer.)

11. Citizens complained to their local elected officials that too many motor vehicles are driven down their residential street, O'Keefe Street, because people use it to take a short cut around a busy intersection. An industrious resident noted that 111 residential units are served by O'Keefe Street, and he obtained traffic data from a State of Virginia traffic engineer who counted 1729 motor vehicles using O'Keefe Street in a 24 hour period. Not only did the resident draw a picture of the data but he formulated the problem carefully by first defining the random variable, V, to be the volume of traffic on O'Keefe Street. According to the *Institute of Transportation Engineers (ITE) Trip Generation Manual*, V is normally distributed with mean, 1016, and standard deviation, 364. What is the probability that the citizens are wrong and the traffic engineer is right by saying that the volume of traffic on O'Keefe Street is not unusual? In other words, what is the probability that the observed volume of traffic would have equaled or exceeded 1729, if O'Keefe Street were a typical residential street?

12. A certain amateur musician's checking account fluctuates with time due to automatic debits and deposits. Let X be a random variable which measures the amount of money which is in the checking account and suppose $X \sim U(\$3000, \$7000)$. One night at the local tavern, the musician capriciously wrote a \$6,000 check to pay for an accordion which someone brought in to sell. What is the probability that the check will bounce?

13. Let U be distributed as N(5,4).

    (a) Find P(U$\leq$ 3)=

    (b) $P(3U + 2 > 20) =$

    (c) $E[3U + 2]$=

    (d) $var(3U + 2)$=

    Find a such that it makes each of the following statements true where $T \sim U(20, 45)$:

14. $P(T \geq a) = .6$

15. $P(25 \leq T \leq a) = .7$

16. Age at diagnosis for each of 20 patients under treatment for meningitis was given in a paper "Penicillin in the Treatment of Meningitis"(*J. Amer. Med. Assoc.* (1984): 1870-1874). The ages reported in years were as follows:

    18, 18, 25, 19, 23, 20, 69, 18, 21, 18, 18, 20, 18, 18, 20, 18, 19, 28, 17, 18
    ($\bar{x} = 22$ and $s = 11$). Let W be the age at diagnosis, and let us assert that
    $W \sim N(\bar{x}, s^2)$. What are:

    (a) $E[W]$=

    (b) $var(W)$=

    (c) $P(W < 20)$=

17. A manufacturer builds an automobile such that it will last six years with a standard deviation of 1.5 years. The manufacturer offers a three year warranty on its product. Assume that the life of the car follows a Normal distribution. What proportion of the manufactured cars will fail under warranty?