

ANSWER KEY

Stat 51: Homework 6

1. A random sample of 10 observations is drawn from a normal population. The sample mean, $\bar{x} = 4$ and the sample variance, $s^2 = .5$. Find the 95% confidence interval for the mean, μ .

Answer 1 95% C.I. is (3.4914,4.505)

2. The demand for meat at a grocery store for a weekend is normally distributed with mean equal to 4900 pounds and standard deviation equal to 500 pounds. Find the 90% confidence interval for the expected demand.

$$\begin{aligned}\text{Answer 2 } a &= \mu - \frac{\sigma}{\sqrt{n}} z_{\frac{\alpha}{2}} = 4900 - \frac{500}{\sqrt{1}} 1.64 = 4080 \\ b &= \mu + \frac{\sigma}{\sqrt{n}} z_{\frac{\alpha}{2}} = 4900 + \frac{500}{\sqrt{1}} 1.64 = 5720\end{aligned}$$

3. Given the sample, $S = \{3 \ 5 \ 1 \ 5 \ 5 \ 1 \ 5 \ 4 \ 0 \ 4 \ 5 \ 4 \ 5 \ 6 \ 2 \ 6 \ 1 \ 4 \ 5 \ 6\}$

(a) Find the 95% confidence interval of $E[\bar{x}]$. (Hint: use Student's t-distribution).

Answer 3 95% CI is: (2.974,4.7525)

(b) Draw a histogram of S .

(c) Draw 95% confidence interval of $E[\bar{x}]$ on the histogram.

4. A day care class consists of 12 children of ages three and four years old. We will refer to this class as a sample. The weight in pounds of each child is:

$$S = \{30 \ 50 \ 40 \ 40 \ 45 \ 35 \ 35 \ 25 \ 40 \ 35 \ 40 \ 30\}$$

Find the 95% confidence interval about the population mean.

Answer 4

95% CI is: (32.7026,41.46406)

5. From a list of 90,000 farmers who operate a farm in Ohio, a sample of 2,000 is drawn, but only 64% of them cooperate in giving an interview. Let X be the number of useful interviews. Find two numbers, a and b , which are symmetric about X such that $P(a \leq X \leq b) = 95\%$. (Hint: There is only one state, so $n=1$.) Note: $z_{.025} = 1.96$

$$\begin{aligned}\text{Answer 5 } a &= E[X] - \frac{\sqrt{npq}}{1} z_{\frac{\alpha}{2}} = 1280 - \frac{\sqrt{460.8}}{1} 1.96 = 1238 \\ b &= E[X] + \frac{\sqrt{npq}}{1} z_{\frac{\alpha}{2}} = 1280 + \frac{\sqrt{460.8}}{1} 1.96 = 1322 \\ P(1238 \leq X \leq 1322) &= .95\end{aligned}$$

6. The grade point averages (GPA) of a sample of 100 students were obtained. Denote the GPA of a student by X_i . From the data, it was found that $\bar{x} = 3.5$ and $s = .5$. Find the 90% confidence interval about the population mean.

Answer 6 90% CI is: (3.41,3.58)

7. Find $z_{\frac{\alpha}{2}}$ for $\alpha = .08$

Answer 7 $z_{\frac{\alpha}{2}} = 1.75$

8. The following sample of 16 measurements was selected from a population that is approximately normally distributed:

$S = \{91\ 80\ 99\ 110\ 95\ 106\ 78\ 121\ 106\ 100\ 97\ 82\ 100\ 83\ 115\ 104\}$

(a) Construct a 80% confidence interval for the population mean.

Answer 8 80% CI is: (93.692,102.183)

(b) Interpret the meaning of this confidence interval for your STAT51 professor.

Answer 9 *We are 80% confident that the population mean is in the confidence interval (93.6978,102.177). Or, if 100 identical and independent experiments were to be conducted, then on the average 80% of them will contain the population mean.*

(c) The 95% confidence interval is: (91.19876,104.6762). Explain why the 80% confidence interval is narrower than the 95% confidence interval.

Answer 10 *The 80% CI is shorter than the 95% CI, because $t_{15,.1} = 1.341$ whereas $t_{15,.025} = 2.131$. Heuristically, the 95% CI must be wider than the 80% CI, because, since we are requiring more confidence to capture the population mean, it is necessary to cover a wider area.*

9. A random sample of 49 observations is drawn from a normal population with mean equal to 50 and $\sigma = 15$. Find c such that $P(\bar{x} \leq c) = .89$.

Answer 11 $c = 52.49$

10. Suppose T_{10} is a Student's t distribution with 10 degrees of freedom. Find t_0 such that $P(T_{10} \leq t_0) = .05$.

Answer 12 $t_0 = -1.812$