Homework 6

1. A random sample of 10 observations is drawn from a normal population. The sample mean, $\bar{x} = 4$ and the sample variance, $s^2 = .5$. Find the 95% confidence interval for the mean, μ.

2. The demand for meat at a grocery store for a weekend is normally distributed with mean equal to 4900 pounds and standard deviation equal to 500 pounds. Find the 90% confidence interval for the expected demand.

3. Given the sample, $\mathcal{S} = \{3\ \ 5\ \ 1\ \ 5\ \ 5\ \ 1\ \ 5\ \ 4\ \ 0\ \ 4\ \ 5\ \ 4\ \ 5\ \ 6\ \ 2\ \ 6\ \ 1\ \ 4\ \ 5\ \ 6\}$

   (a) Find the 95% confidence interval of $E[\bar{x}]$. (Hint: use Student's t-distribution).

   (b) Draw a histogram of $\mathcal{S}$.

   (c) Draw 95% confidence interval of $E[\bar{x}]$ on the histogram.

4. A day care class consists of 12 children of ages three and four years old. We will refer to this class as a sample. The weight in pounds of each child is:

   $\mathcal{S} = \{30\ \ 50\ \ 40\ \ 40\ \ 45\ \ 35\ \ 35\ \ 25\ \ 40\ \ 35\ \ 40\ \ 30\}$

   Find the 95% confidence interval about the population mean.

5. From a list of 90,000 farmers who operate a farm in Ohio, a sample of 2,000 is drawn, but only 64% of them cooperate in giving an interview. Let X be the number of useful interviews. Find two numbers, a and b, which are symmetric about X such that $P(a \leq X \leq b) = 95\%$. (Hint: There is only one state , so n=1.) Note: $z_{.025} = 1.96$

6. The grade point averages (GPA) of a sample of 100 students were obtained. Denote the GPA of a student by X_i. From the data, it was found that $\bar{x} = 3.5$ and $s = .5$. Find the 90% confidence interval about the population mean.

7. Find $z_{\frac{\alpha}{2}}$ for $\alpha = .08$

8. The following sample of 16 measurements was selected from a population that is approximately normally distributed:
   $\mathcal{S} = \{91\ 80\ 99\ 110\ 95\ 106\ 78\ 121\ 106\ 100\ 97\ 82\ 100\ 83\ 115\ 104\}$

   (a) Construct a 80% confidence interval for the population mean.

   (b) Interpret the meaning of this confidence interval for your STAT51 professor.

   (c) The 95% confidence interval is: (91.19876,104.6762). Explain why the 80% confidence interval is narrower than the 95% confidence interval.

9. A random sample of 49 observations is drawn from a normal population with mean equal to 50 and $\sigma = 15$. Find c such that P($\bar{x} \leq c$)=.89.

10. Suppose T_{10} is a Student's t distribution with 10 degrees of freedom. Find t_0 such that $P(T_{10} \leq t_0) = .05$.