A N S W E R K E Y

Stat 51: Homework 7

1. A manufacturer of sports equipment has developed a new synthetic fishing line that he claims has a mean braking strength of 15 pounds with a standard deviation of .5 pounds. Test the hypothesis that $\mu = 15$ against the alternative that $\mu \neq 15$ if a random sample of 50 lines is tested and found to have a mean braking strength of 14.8 pounds. That is, should the null hypothesis be rejected or accepted? Use a .01 level of significance.

Answer 1 *This question is ambiguous. Is the standard deviation the population standard deviation or the sample standard deviation?*

Let us assume as in almost all practical cases that the descriptive statistics were obtained from experimental data; therefore, the standard deviation is the sample standard deviation.

(a) $\alpha = .01$ *Two-sided test* $\rightarrow \frac{\alpha}{2} = .005$ *n-1=49*

(b) $t_{49,.005} = 2.679952$

(c) $T = \frac{14.8-15}{\frac{.5}{\sqrt{50}}} = -2.828427$

(d) *Is* $T = -2.828427 < -2.679952$? *or* $-2.828427 > 2.679952$ *Yes*

(e) *Reject null hypothesis.*

2. A random sample of n=25 observations is drawn from a normal population with standard deviation equal to 9. The sample mean was found to be, $\bar{x} = 16.5$ Test the hypothesis: $H_o : \mu = 15$ vs $H_1 : \mu > 15$ at the .05 level of significance.

Answer 2 (a) $\alpha = .01$ *One-sided test*

(b) $z_{.05} = 1.64$

(c) $Z = \frac{16.5-15}{\frac{9}{\sqrt{25}}} = .833$

(d) *Is* $Z = .833 > 1.64$ *No*

(e) *Cannot reject null hypothesis.*

3. According to the American Medical Association, approximately 50% of all U. S. physicians whose ages are under 35 are known to be women. Four samples of 15 physicians were interviewed each. Let X_i be the number of women that were found in the i^{th} random sample of 15 physicians whose ages are less than 35. The number of women in each of the four samples were counted and it was found that $\bar{x} = 5$ and $s^2 = \frac{2}{3}$. What is the 95% confidence interval about μ? (Hint: Let X_i be approximately distributed as a normal, $N(\mu, \sigma^2)$. Use n=4 in the formula for the lower and upper limits and the t distribution).

Answer 3 (a) $\alpha = .05$ *so* $\frac{\alpha}{2} = .025$ *and n-1=3.*

(b) $t_{3,.025} = 3.182$

(c) *lower limit*

$a = \bar{x} - \frac{s}{\sqrt{n}} t_{n-1,\frac{\alpha}{2}} = 5 - \frac{\sqrt{\frac{2}{3}}}{\sqrt{4}} 3.182 = 3.7009$

(d) *upper limit*
$b = \bar{x} + \frac{s}{\sqrt{n}} t_{n-1,\frac{\alpha}{2}} = 5 + \frac{\sqrt{\frac{2}{3}}}{\sqrt{4}} 3.182 = 6.299$

(e) *95% CI is: (3.7009,6.299)* ***Parentheses and comma are required for correct notation.***

4. Continuation of problem 3. Given that $\bar{x} = 5$ and $s^2 = \frac{2}{3}$ from the 4 samples of 15 physicians each, test the hypothesis that $\mu = 7.5$ against the alternative that $\mu \neq 7.5$ at the .05 level of significance. (Hint: Use the test where σ^2 is unknown and use n=4 in the formula of the test statistic).

Answer 4 (a) $\alpha = .05$ *Two-sided test* $\rightarrow \frac{\alpha}{2} = .025$ *n-1=3*

(b) $t_{3,.005} = 3.182$

(c) $T = \frac{5-7.5}{\frac{\sqrt{\frac{2}{3}}}{\sqrt{4}}} = -6.12$

(d) *Is* $T = |-6.12| > 3.182$ *Yes*

(e) *Reject null hypothesis.*

5. Given the sample, $\mathcal{S} = \{3\ 5\ 1\ 5\ 5\ 1\ 5\ 4\ 0\ 4\ 5\ 4\ 5\ 6\ 2\ 6\ 1\ 4\ 5\ 6\}$. Test the hypothesis: $H_0 : \mu = 3$ vs $H_1 : \mu \neq 3$ at the .05 level of significance.

Answer 5 $\bar{x} = 3.85$ $s = 1.8715$

(a) $\alpha = .05$ *Two-sided test* $\rightarrow \frac{\alpha}{2} = .025$ *n-1=19*

(b) $t_{19,.025} = 2.093$

(c) $T = \frac{3.85-3}{\frac{1.8715}{\sqrt{20}}} = 2.03116$

(d) *Is* $T = |2.03| > 2.093$ *No*

(e) *Cannot reject null hypothesis.*

6. Let t_0 be a specific value of t. Find t_0 such that $P(t \geqslant t_0)$=.01 where df=14.

Answer 6 *We note that* $P(T_{14} \geqslant t_0) = .01 \rightarrow t_{14,.01} = 2.624494$. *A picture of the t distribution should have been drawn as a guide.*

7. The temperature in Fahrenheit of six ovens which are used in a manufacturing process of cintering ferrite yokes for television sets were recorded
$\mathcal{S} = \{1325\ 1323\ 1340\ 1331\ 1328\ 1323\}$. Test the hypothesis : $H_0 : \mu = 1325$ vs $H_1 : \mu \geqslant 1325$ at the .05 level of significance when the p-value is 0.1279207.

Answer 7 *Easy way:* $\alpha < p - value$ *hence cannot reject* H_0.

The hard and long way: $\bar{x} = 1328.33$ *s=6.501282.*

(a) $\alpha = .05$ *Two-sided test and variance is unknown hence T test statistic is used.*

(b) $t_{5,.025} = 2.5706$

(c) $T = \frac{1328.333-1325}{\frac{6.501282}{6}} = 1.255901$

(d) *Is* $T = 1.2559 < -2.57$ *or* $1.2559 > 2.57$ *? NO*

(e) *Cannot reject null hypothesis.*

8. A random sample of 20 observations selected from a normal population produced $\bar{x} = 72.6$ and $s^2 = 19.4$. Test $H_0 : \mu = 80$ vs $H_1 : \mu < 80$. Use $\alpha = .05$

Answer 8 *(a)* $\alpha = .05$ *One-sided test n-1=19*

(b) $t_{19,.05} = 1.729133$

(c) $T = \frac{72.6-80}{\frac{\sqrt{19.4}}{\sqrt{20}}} = -7.513$

(d) Is $T = -7.513 < -1.729$ *Yes*

(e) Reject null hypothesis.

9. A professor of social work observed that the attitude of students with regard to social issues seems to change in the span of time from senior year in college to receiving a masters degree. To test his observation, the professor asked 10 students who were in their senior year the question, "Is there too little space exploration?". The answers were reported using a five point scale, 1 strongly disagree to 5 strong agree, they appear in Table 9. All 10 students were women. Test the null

Student	Before Bachelor's Degree x	After Masters Degree y	Difference d=x-y
1	2	5	-3
2	3	2	1
3	2	4	-2
4	2	3	-1
5	2	3	-1
6	3	5	-2
7	5	3	2
8	3	5	-2
9	2	2	0
10	1	2	-1
mean	2.5	3.4	-0.636
std	1.166667	1.6	1.689

hypothesis that going to graduate school changes one's attitude on the issue of funding the space program against the alternative that going to graduate school does not cause a change in attitude at a level of significance of .05.

Answer 9 *This is a paired difference test. See the table for the column of differences. Test:* $H_0 : d = 0$ *vs* $H_1 : d \neq 0$ *at* $\alpha = .05$

(a) $\alpha = .05$ *Two-sided test* $\rightarrow \frac{.05}{2} = .025$ *. Degrees of freedom=n-1=9*

(b) $t_{9,.025} = 2.26216$

(c) $T = \frac{-0.636-0}{\frac{1.689}{\sqrt{11}}} = -1.249$

(d) Is $T = -1.249 < 2.26216$ *or is* $-1.249 > 2.26216$*? No*

(e) Cannot reject null hypothesis. Going to graduate school does not appear to change the political sentiments of students.

10. STAT51 students counted the number of e's for an experiment in counting on the first day of class. The average number of e's from the 47 students is 218.957 with a sample standard deviation of 71.143. The hypothesis is that the class average equals the true value of 315 versus the alternative hypothesis that the class average is less than the true value. Use a .05 level of significance and only the p-value=2.25×10^{-12} to decide whether to reject or not reject the null hypothesis.

Answer 10 $.05 = \alpha >$ *p-value*=$2.25 \times 10^{-12} \rightarrow$ *reject* H_0.

11. Construct a 90% confidence interval about the mean for the number of e's which the STAT51 students in the previous problem counted. Why is the confidence interval for the STAT51 students so far away from the true number of e's?

Answer 11 *(a)* $\bar{x} = 218.957$ *s=71.143 n=47*

(b) $\alpha = .10$ *so* $\frac{\alpha}{2} = .05$ *and n-1=46.*

(c) $t_{46,.05} = 1.67866$

(d) lower limit
$a = \bar{x} - \frac{s}{\sqrt{n}} t_{n-1,\frac{\alpha}{2}} = 218.957 - \frac{71.143}{\sqrt{47}} 1.67866 = 201.53$

(e) upper limit
$b = \bar{x} + \frac{s}{\sqrt{n}} t_{n-1,\frac{\alpha}{2}} = 218.957 + \frac{71.143}{\sqrt{47}} 1.67866 = 236.37$

(f) 90% CI is: (201.53,236.37) ***Parentheses and comma are required for correct notation***. $315 \notin (201, 236) \rightarrow$ *reject* H_0