A N S W E R K E Y

Homework 8

1. An amateur gardener wanted to discover whether one fertilizer is better than another when applied to his tomato plants. Having taken a statistics course in the design of experiments, this gardener knew that conditions for all 11 of his tomato plants must be the same so that the effects of applying the two different fertilizers will not be confounded by something else. Therefore, he planted the 11 tomato plants of the same hybrid in the same plot with the same exposure to sunlight, etc. He applied fertilizer A to five tomato plants which were chosen at random, and he applied fertilizer B to six other tomato plants. At harvest, the average weight of a tomato from each plant was obtained and is tabulated below:

Average Weight of a Tomato	
Fertilizer A	Fertilizer B
29.9 oz	26.6 oz
11.4	23.7
25.3	28.5
16.5	14.2
21.1	17.9
.	24.3

Is there a difference in the yield due to the different fertilizers at the .05 level of significance? (Hint:Test the hypothesis:$H_0 : \mu_1 - \mu_2 = 0$ vs $H_1 : \mu_1 - \mu_2 \neq 0$).

	A	B
n	5	6
$\bar{x}$	20.84	22.53
s	7.245	5.43

Answer 1 *(a)* $\alpha = .05$ $\frac{\alpha}{2} = .025$ $\nu = n_1 + n_2 - 2 = 5 + 6 - 2 = 9$ *Two-sided test.*

(b) $t_{9,.025} = 2.262$

(c) $S_p^2 = \frac{4(7.245^2)+5(5.43^2)}{9} = 39.72504$ $S_p = 6.30278$

(d) $T = \frac{\mu_1-\mu_2-d_0}{s_p\sqrt{1/n_1+1/n_2}} = \frac{20.84-22.53-0}{6.30278\sqrt{1/5+1/6}} = -.44$

(e) Is $T = -.44 < -2.262$*? No*

(f) Cannot reject null hypothesis. Fertilizers apparently do not make a difference in the yield of tomatoe plants.

2. A farmer claims that the average yield of corn of variety A exceeds the average yield of variety B by at least 12 bushels per acre. To test this claim, 20 one acre plots of each variety are planted and grown under similar conditions. Variety A yielded, on the average, 86.7 bushels per acre with a sample standard deviation of 6.28 bushels per acre, while variety B yielded, on the average, 77.8 bushels per acre with a sample standard deviation of 5.61 bushels per acre. Test the farmer's claim using a .05 level of significance. (Hint: Test the hypothesis: $H_0 : \mu_1 - \mu_2 = 12$ vs $H_1 : \mu_1 - \mu_2 > 12$).

	A	B
n	20	20
$\bar{x}$	86.7	77.8
s	6.28	5.61

Answer 2 *(a)* $\alpha = .05$ *One-sided test.* $n_1 + n_2 - 2 = 20 + 20 - 2 = 38$

(b) $S_p^2 = \frac{19(6.28)^2+19(5.61)^2}{38} = 35.456\ S_p = 5.9544$

(c) $t_{38,.05} = 1.686$

(d) $T = \frac{\bar{x}_1-\bar{x}_2-d_0}{S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{86.7-77.8-12}{5.954\sqrt{1/20+1/20}} = -1.646466$

(e) Is $T = -1.646466 > 1.680$*? No*

(f) Cannot reject null hypothesis; the farmer's claim is probably exaggerated.

(g) (p-value=0.05395735; therefore. we cannot reject H_0*. Suppose that* $d_0 = 5$*, then T=2.07 and p-value=.022, then a case can be made that there is at least a 5 bushel difference in the yields.)*

(h) Let us construct 95% one-sided confidence interval about $\mu_1 - \mu_2$

i. $\alpha = .05\ n_1 + n_2 - 2 = 20 + 20 - 2 = 38$

ii. $S_p^2 = \frac{19(6.28)^2+19(5.61)^2}{38} = 35.456\ S_p = 5.9544$

iii. $t_{38,.05} = 1.686$

iv. a=$\bar{x}_1 - \bar{x}_2 - S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}t_{38,.05} = 86.7 - 77.8 - 5.954\sqrt{1/20 + 1/20}1.686 = 5.72$

v. b=∞

vi. The 95% one-sided confidence interval is $(5.72, \infty)$*. Is* $12 \in (5.72, \infty)$*? Yes, therefore the farmer's claim is not substantiated by the data. Is* $5 \in (5.72, \infty)$*? No, therefore there appears to be at least a 5 bushel difference. You can tell that I favor confidence intervals over the method of testing hypotheses, because you can see the range over which our best guess for the population parameter is located. In this problem, it appears that a difference of about* $5 \approx 5.72$ *bushels is the more likely difference than the farmer's expert opinion.*

3. Let $\bar{x}_1 = 61,600$, $n_1 = 12$, and $s_1 = 3300$. Let $\bar{x}_2 = 64,000$, $n_2 = 12$, and $s_2 = 3500$. Assume that $\sigma_1^2 = \sigma_2^2$. Test the hypothesis:

$H_0 : \mu_1 - \mu_2 = 0$ vs $H_1 : \mu_1 - \mu_2 \neq 0$ at $\alpha = .05$

	#1	#2
n	12	12
$\bar{x}$	61600	64000
s	3300	3500

Answer 3 *(a)* $\alpha = .05$ *Two-sided test.* $\frac{\alpha}{2} = .025$ $n_1 + n_2 - 2 = 12 + 12 - 2 = 22$

(b) $S_p^2 = \frac{11(3300)^2 + 11(3500)^2}{22} = 11570000$ $S_p = 3401.47$

(c) $t_{22,.025} = 2.073873$

(d) $T = \frac{\bar{x}_1 - \bar{x}_2 - d_0}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{61600 - 64000}{3401.47\sqrt{1/12 + 1/12}} = -1.728$

(e) Is $|T| = 1.728 > 2.073873$*? No*

(f) Cannot reject null hypothesis.

(g) (p-value=.09794569; therefore. we cannot reject H_0*.*

4. A Cuckoo will lay eggs in the nests of other birds. It is claimed by some biologists that the size of an egg which a Cuckoo will lay conforms to the size of the egg of a warbler or of a wren depending in which nest the Cuckoo lays her egg. The following descriptive statistics given in Table 1 were obtained for Cuckoo eggs which were found in randomly sampled nests of warblers and wrens. Test the hypothesis: $H_0 : \mu_1 - \mu_2 = 0$ vs $H_1 : \mu_1 - \mu_2 \neq 0$ at a level of significance of .01 where #1 designates the population of warbler nests and #2 designates the population of wren nests.

Table 1:

	#1	#2
n	29	35
$\bar{x}$	22.20 mm	21.12 mm
s	.65 mm	.75 mm

Answer 4 *(a)* $\alpha = .01$ $\frac{\alpha}{2} = .005$ $\nu = n_1 + n_2 - 2 = 29 + 35 - 2 = 62$ *Two-sided test.*

(b) $t_{62,.005} = 2.660$

(c) $S_p^2 = \frac{28(.65^2) + 34(.75^2)}{62} = 0.4992742$ $S_p = .706593$

(d) $T = \frac{\mu_1 - \mu_2 - d_0}{s_p\sqrt{1/n_1 + 1/n_2}} = \frac{22.20 - 21.12 - 0}{.706593\sqrt{1/29 + 1/35}} = 6.0869$

(e) Is $T = 6.0869 < -2.660$*? or* $6.0869 > 2.660$ *Yes*

(f) *Reject null hypothesis. The sizes of Cuckoo eggs do conform to the size of the host species eggs.*

5. Residents usually welcome the installation of speed humps on their streets in order to reduce speeding traffic whereas traffic engineers despise speed humps because by impeding cut-through traffic they divert cut-through traffic onto the arterial routes. The engineers recorded the speeds before and after the installation of speed humps to see if they have an effect. A tabulation of speeds on some selected streets are given below. At a level of $\alpha = .10$, can the engineers conclude that speed humps on the average reduce the speed of traffic by 10 mph?

Street	Before	After
Northborne Drive	45 mph	33 mph
Kings Park Drive	50	27
Hollinger Avenue	43	28
Springhaven Drive	40	30

Answer 5 (a) $\bar{d} = 15$ $s_d^2 = \frac{\sum(d_i - \bar{d})^2}{n-1} = 32.66667$ $s_d = 5.715476$

(b) $\alpha = .10$ *n-1=3*

(c) $t_{3,.10} = 1.63774$

(d) $T = \frac{\bar{d} - d_0}{\frac{s_d}{\sqrt{n}}} = \frac{15-10}{\frac{5.715}{4}} = 1.74978$

(e) *Is* $T = 1.74978 > 1.63774$*? Yes*

(f) *Reject null hypothesis. Speed humps do reduce speed of traffic.*

6. A company bakes computer chips in two ovens, A and B. Chips are randomly assigned to an oven and hundreds of chips are baked each hour. The percentage of defective chips are tabulated below. For example, oven A produced chips which were 45% defective during hour 1. Does there appear a difference between oven A and oven B with respect to the rate of producing defective chips at a 95% level of significance?

	1	2	3	4	5	6	7	8	9
A	45%	32	34	31	35	37	31	30	27
B	36%	37	33	34	33	32	33	30	24
d	9%	-5	1	-3	2	5	-2	0	3

Answer 6 *Test the hypothesis:* $H_0 : \mu_D = 0$ *vs* $H_1 : \mu_D \neq 0$ *where* $d = A_i - B_i$*Use* $\alpha = .05$

(a) *Compute differences as shown in row d.*

(b) $\bar{d} = 1.1111$ $s_d^2 = \frac{\sum(d_i - \bar{d})^2}{n-1} = 18.3611$ $s_d = 4.284$

(c) $\alpha = .05$ $\frac{\alpha}{2} = .025$ *n-1=8*

(d) $t_{8,.025} = 2.306$

(e) $T = \frac{\bar{d}-0}{\frac{s_d}{\sqrt{9}}} = 0.778$

(f) *Is* $T = 0.778 < -2.306$ *or* $.778 > 2.306$*?No*

(g) *Cannot reject null hypothesis. Oven are producing defective chips at the same rate.*