Open Book — Open Notes

**Name**:

A N S W E R K E Y

# STAT 111: Mock Mid-Term Examination

1. (5pts) From a list of 90,000 farmers who operate a farm in Ohio, a sample of 2,000 is drawn, but only 64% of them cooperate in giving an interview. It is necessary to obtain at least 1,300 useful interviews, in order to produce a reliable estimate of the number of hogs. What is the probability that 2,000 is large enough for the size of the sample, i.e. find $P(X \geq 1300)$. (Hint: use z-score; there is only one state).

   $X \sim b(2000, .64)$ $E[X] = np = 1280$ $var(X) = npq = 460.8$

   $P(\frac{X-1280}{\sqrt{460.8}} \geq \frac{1300-1280}{21.46}) = P(z \geq .93169) = 1 - P(z \leq .93) = 1 - (.5 + .3238) = .1762$

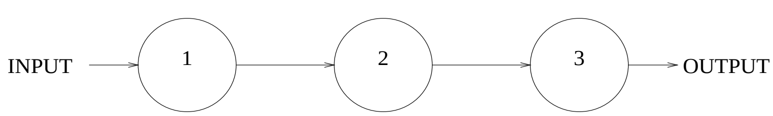
2. (5pts) A day care class consists of 12 children of ages three and four years old. We will refer to this class as the population. The weight in pounds of each child is:

   $\Omega = \{30\ 50\ 40\ 40\ 45\ 35\ 35\ 25\ 40\ 35\ 40\ 30\}$

   Draw a leaf and stem plot of $\Omega$.

| | |
|---|---|
| 0 | |
| 1 | |
| 2 | 5 |
| 3 | 0 0 5 5 5 |
| 4 | 0 0 0 0 5 |
| 5 | 0 |
| 6 | |
| 7 | |
| 8 | |
| 9 | |

3. (5pts)




   Three components which operate independently of each other in series comprise a system. The probability of failure of each component is shown in the following table.

| Component | Probability of Failure |
|---|---|
| 1 | .2 |
| 2 | .15 |
| 3 | .1 |

What is the probability that the system will function without failing?

Let A be the event that component A fails; likewise for B fails; C fails

$P(willwork) = P(A^c \cap B^c \cap C^c) = (.8)(.85)(.9) = .612$

4. (5pts) The average grade for a BIG statistics examination is 75 with a standard deviation of 12. Suppose that the instructor wants to grade the exam on a curve so that 10% of the class will get A's. What is the minimum grade for getting an A on the exam?

   Ans: Let X be a grade. $X \sim N(75, 12^2)$. We need to find c such that: $P(X > c) = .10$ i.e. $P(X \leq c) = P(\frac{X-75}{12} \leq \frac{c-75}{12}) = P(z \leq \frac{c-75}{12}) = .90 = .5 + .4 \rightarrow \frac{c-75}{12} = 1.28$ where 1.28 is found in the table. Therefore, c=90.36

5. Suppose a random sample of n measurements is selected from a population with mean $\mu$=200 and variance $\sigma^2 = 20$. If the value of n=4, what is:

   (a) $E[\bar{x}] = 200$

   (b) $var(\bar{x}) = \frac{\sigma^2}{n} = \frac{20}{4} = 5$

6. (5pts) Let X be a discrete random variable such that X is distributed as a binomial, b(3, $\frac{2}{7}$) distribution. Find: var(X)=$3\frac{2}{7}\frac{5}{7} = \frac{30}{49}$

7. The owner of construction company A bids on jobs so that if awarded the job, company A will make a \$10,000 profit. The owner of construction company B makes bids on jobs, so that if awarded the job, company B will make a \$15,000 profit. Each company describes the probability distribution of the number of jobs the company is awarded per year as shown in the table.

| Company A | | Company B | |
|---|---|---|---|
| 2 | .05 | 2 | .15 |
| 3 | .15 | 3 | .30 |
| 4 | .20 | 4 | .30 |
| 5 | .35 | 5 | .20 |
| 6 | .25 | 6 | .05 |

   (a) ( 5pts) Find the expected number of jobs each will be awarded in a year.

   Company A: 2(.05)+3(.15)+4(.20)+5(.35)+6(.25)=4.6

   Company B: 2(.15)+3(.30)+4(.30)+5(.20)+6(.05)=3.7

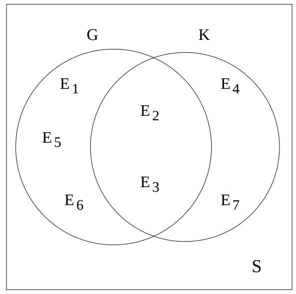
   (b) ( 5pts) What is the expected profit for each company?

   Company A:4.6(\$10000)=\$46,000

   Company B:3.7(\$15000)=\$55,500

8. (5pts) Let U be distributed as N(0,1). Find: P($U \leq -.4$ and $U \geq .4$)=.1554+.1554=.6892

9. Let $P(E_1) = P(E_2) = P(E_5) = \frac{1}{6}$ $P(E_4) = \frac{7}{60}$ $P(E_3) = P(E_6) = \frac{1}{15}$ and $P(E_7) = \frac{1}{4}$.




Find:

(a) (5pts) $P(G \cap K) = P(E_2) + P(E_3) = \frac{1}{6} + \frac{1}{15} = \frac{7}{30}$

(b) (5pts) $P(K|G) = \frac{P(G \cap G)}{P(G)} = \frac{\frac{7}{30}}{\frac{19}{30}} = \frac{7}{19}$

10. Consider the probability distribution shown here:

| k | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|---|---|
| P(X=k) | .02 | .07 | .10 | .15 | .30 | .18 | .10 | .06 | .02 |

Find:

(a) (5pts) $P(-3 \leq X < 3) = .9$

(b) (5pts) E[X]=0

11. (5pts) Let $X \sim b(15, .5)$. Compute: $P(3 \leq X < 8)$. (Normal approximation not acceptable).
$P(X \leq 7) - P(X \leq 2) = .5 - .004 = .496$

12. (5pts) Find a such that it makes the following statement true where $X \sim U(3, 8)$: $P(X \geq a) = .6$
$(8 - a)\frac{1}{5} = .6$ a=5

13. (5pts) Let $Y_1, Y_2, Y_3, Y_4$, and $Y_5$ be independent random variables each of which follows the binomial distribution, b(4,$\frac{7}{9}$) and denote, $\bar{Y} = \frac{Y_1+Y_2+Y_3+Y_4+Y_5}{5}$. Find: var($\bar{Y}$)=$\frac{var(y_i)}{5} = \frac{4(\frac{7}{9})(\frac{2}{9})}{5} = \frac{56}{405}$.

14. Let W, the amount of someone's lottery prize, be a random variable such that:
W=$1000 with probability .2; W=$50 with probability .5; and W=$0 with probability .3

(a) (5pts) Find E[W]=1000(.2)+50(.5)=200+25=225.

(b) (5pts) Find var(W)=$(1000 - 225)^2(.2) + (50 - 225)^2(.5) + (0 - 225)^2(.3) = 150625$.

15. (5pts) A random sample of n=25 observations is drawn from a normal population with mean equal to 10 and standard deviation equal to 16. Find c such that P($\bar{X} \leq c$)=.95. c= $\bar{x} \sim N(10, \frac{256}{25})$ $P(\frac{\bar{x}-10}{\frac{16}{5}} \leq \frac{c-10}{\frac{16}{5}}) = .95$ $\frac{c-10}{\frac{16}{5}} = 1.64$ c=15.248

B5 **BONUS** (10pts) Once upon a time, leaders of the largest real estate developers recognized that the current economy was depressing the new housing market. With adroit campaigning, they persuaded the local politicians to relax the building code just a little bit, claiming that by constructing less expensive houses, they could revitalize the housing market and thereby bring prosperity back to their hometown. A few years later, the cheap plastic piping began to burst in the 80,000 houses which were built under the relaxed building code. Some industrious homeowners successfully brought about a class action suit. They won, but the manufacturers of the plastic claimed that they did not have enough money to replace the piping with copper in all of those houses. The judge therefore ruled that 95% of the houses were in scope of the settlement depending on the age of the piping; the other 5%, well, they were out of luck. The life expectancy of the plastic pipe is normally distributed with mean 10 years and standard deviation of 3 years. What is the cut off age of a house before which the owner can still can get copper piping but after which the house will be out-of-scope? Ans: Let A=age. Find $P(A < c) = .95$ $P(\frac{A-10}{3} \leq \frac{c-10}{3}) = .95 = .5 + .45$ c=14.92